Problem Solving Lesson Plan:
Factors and Greatest Common Factors

Krista Hemmelgarn
Maths 150
Dr. Shafer
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**Content Objectives**
After this lesson is completed, students will be able to find the factors of given whole number values. Students will also be able to identify the greatest common factor between two or more whole numbers.

**Process Objectives**
The process standard that is incorporated in the following lesson is the representation standard. Throughout my lesson, especially in the activities, many types of representation are used. Some types include: modeling, tables, and pictures that all represent the content objectives and learning goals this lesson is going to reach.

**Learning Goals**
Indiana State Standards – Sixth Grade:
Number Sense
6.1: Students compare and order positive and negative numbers, decimals, fractions, and mixed numbers. They find multiples and factors.
   6.1.7: Find the least common multiple and the greatest common factor of whole numbers. Use them to solve problems with fractions. (example: finding common denominators to add fractions)


**Opener**
Engage students in the following warm-up activity. This activity will trigger a discussion of why different whole numbers can be multiplied together to get the same whole number answer. It will also allow students to observe that the same whole number can be divided by different numbers. This activity will help students realize the idea of factors without teaching it directly.

**Warm-Up Activity:**
Complete these multiplication and division problems.

\[
\begin{align*}
6 \times 6 &= \underline{\phantom{1}36\phantom{0}} & 18 \times 2 &= \underline{\phantom{1}36\phantom{0}} & 12 \times 3 &= \underline{\phantom{1}36\phantom{0}} & 9 \times 4 &= \underline{\phantom{1}36\phantom{0}} & 36 \times 1 &= \underline{\phantom{0}36}\phantom{0} \\
48 \div 1 &= \underline{\phantom{1}48\phantom{0}} & 48 \div 6 &= \underline{\phantom{1}8\phantom{0}} & 48 \div 2 &= \underline{\phantom{1}24\phantom{0}} & 48 \div 8 &= \underline{\phantom{1}6\phantom{0}} & 48 \div 4 &= \underline{\phantom{1}12\phantom{0}}
\end{align*}
\]

**Answers**

\[
\begin{align*}
6 \times 6 &= \underline{36} & 18 \times 2 &= \underline{36} & 12 \times 3 &= \underline{36} & 9 \times 4 &= \underline{36} & 36 \times 1 &= \underline{36} \\
48 \div 1 &= \underline{48} & 48 \div 6 &= \underline{8} & 48 \div 2 &= \underline{24} & 48 \div 8 &= \underline{6} & 48 \div 4 &= \underline{12}
\end{align*}
\]

Class Discussion Questions
1. Why are the answers to the multiplication problems the same whole number?
2. Why can the same whole number, 48, be divided by different numbers?
Possible Answers to Class Discussion Questions:

1. **Students may have answers like:** The answers to all of the multiplication problems are the same because different numbers can go into 36. Or the students may say that 36 has many different factors.

2. **Students may have answers like:** Different numbers can divide 48 evenly because there are many numbers that go into 48. The number 48 also has many factors.

The answers above suggest the students do have some prior knowledge of factors. This activity and lesson will provide formal explanations of the terms factor and greatest common factor. It will also clearly state the procedure to find the greatest common factor.

**Activities**

**Clock Activity**

This activity effectively allows students to explore factors. It is an exploration activity that shows students how factors work and what they mean, without directly teaching them. In this activity students are given 4 different clock faces with different numbers of dots: 12, 13, 14, and 15. The students will designate a start dot and label it start. Each clock corresponds with a 2-column table. Each column is labeled “jump size” and “number of jumps”. Each table lists the possible jump sizes. For example, the clock with 15 dots could have jump sizes of 1 – 15. The students must experiment to see which jump sizes gives them an even number of jumps back to the “start” dot. If there is an even number then the students will record yes and the number of jumps in the column. If there is not an even number of jumps then they will record no in the box. To start the activity, the teacher will use the clock with 15 dots as an example.

After the activity is complete the students will then be instructed to list the factors of 12, 13, 14, and 15 from the observations and the table they just completed. After students make the list of factors, they will be asked to find the greatest factor 12 and 13 have in common, or 14 and 15, and so on. After the students have done this part of the activity, explain to them that they found the greatest common factor. Also, explain to them why some jump sizes ended up on the start dot and why some did not. The explanation is the factors can only be whole numbers. In this activity you could not have a partial number of jumps. The number of jumps had to be a whole number that allowed you to end back where you began.

**Challenge Questions:**

After all steps of the activity are complete, ask the students to list all the factors of 30. Then ask them to compare the list to the factor lists of 12, 13, 14, and 15. Then ask them the following questions:

1. Find the GCF between 15 and 30?
2. Find the GCF between 12, 15, and 30?
3. Find the GCF between 12, 13, 15, and 30?
4. Find the GCF between 12, 13, 14, 15, and 30?
Lastly, review with the students the terms they learned and how to find them.

1. **Factors**: Whole numbers that can be multiplied together to get another whole number. For example, in the activity the “number of jumps” and the “jump size” are the factors of the “number of dots”. Factors must be whole numbers because you could not have a partial jump to end back on the start dot.

2. **Greatest Common Factor (GCF)**: Is the greatest factor two or more numbers have in common. The GCF can be found by listing the factors of each number and finding which factor is the greatest and the numbers have in common.

**Closer**
Complete these problems (Don’t forget to show your work!):

1. George wishes to create a rectangular garden in his backyard. He knows that his garden needs to be a total of 36 square feet. List all possible dimensions that George’s garden can have. Or in other words, make a list of factor pairs that fit the description of the garden.

2. George’s next-door neighbor, Sally, wishes to also make a garden. Sally’s garden must be 96 square feet. Make a list of all possible dimensions Sally’s garden can have.

3. The two gardens are going to have to share a common side. The side that is shared must be as long as possible. What is the length of the side that is shared between the two gardens?

4. Explain, in the correct terms, how you solved this problem using the information you learned in the lesson. Show all work leading up to your answer.

After students have completed the problem, involve them in a class discussion. Review with them what was learned in the lesson: what a factor is and how to find a GCF between two or more whole numbers.

* See handout for possible student answers.

**Assessment**
Ask the students to fill out an exit ticket with the following questions:

1. Explain in your own words the terms factor and greatest common factor.

2. Explain or show how you would find the greatest common factor between 50 and 45.

3. List at least one concept or topic from today’s lesson that you are confused about or do not understand completely.
**Diverse Learners**

My lesson plan meets the needs of diverse learners in many ways. There are a few diversity issues that may be present in a classroom doing this particular lesson. One diversity issue is the ability levels of the students. Some students may understand this activity and lesson completely and others may not understand the concepts at all. To help those students struggling with the concepts, I broke the students apart into groups so there are mixed ability levels in each group. This not only will help the students who are struggling, but will also help the students at a higher ability level. In my opinion, students learn better when they are given the opportunities to express what they are learning to another individual. By mixing the ability levels in the groups, higher-level students are given those opportunities to share their knowledge with their peers. The students who are struggling will also get the opportunity to learn directly from their peers.

A scenario that illustrates this type of diversity in the classroom would be when a student moves faster at the activity then the other students. This student would need to be challenged more than the other students. This is why I chose to put a challenge at the end of the clock activity. The challenge question pushes the student to think a little more about factors and how to find factors of larger numbers. The student is also able to practice finding the greatest common factor between more than two numbers.

Another scenario that may occur in this lesson is a student may fall behind while completing the activity. A student may not understand what is being asked of them and struggle to keep up with the other students. To help this situation, I would put these students in groups that would make them more successful. Their peers would be able to help them through the activity, and if assistance from me, the teacher, is needed I will be there to help.

**Student Account of the Problem Solving Lesson**

**Opener**

Students will complete the warm-up activity that will test their knowledge about factors. They will complete the two questions that will trigger a class discussion about why the answers to all the multiplication problems are the same and why 48 can be divided by many different whole numbers. This warm-up activity serves as a nice transition into the main activity.

**Clock Activity**

Students will be put into groups of 4 and 5. They will complete the activity by filling out the activity sheets. This activity allows them to explore factors and they are able to see what factors each of the numbers have in common. This activity then leads into a class discussion of what a factor is and how to find a greatest common factor, which are the content objectives and learning goals of this lesson.

**Closer**

Through the garden problem students are able to use a real life example to apply these concepts. The first find the factors of two numbers, 36 and 96. They then have to
find the greatest common factor because the side that is shared by the two gardens must be the longest possible side. This problem uses a real life situation to connect to the concepts being taught in this lesson.

**Assessment**

Students answer the questions to the best of their abilities. They answer the last question, which asks what was confusing about the lesson and what do they not understand. This allows the teacher to reflect about what the students really learned. This will help the teacher in further lessons throughout the year.

**Sources**


Dr. Kathryn G. Shafer. November 9, 2011. Personal interview.
Name: _______________________________________

Warm-Up Activity

1. Complete these multiplication and division problems.

   \[6 \times 6 = \underline{\phantom{0}}\quad 18 \times 2 = \underline{\phantom{0}}\]

   \[12 \times 3 = \underline{\phantom{0}}\quad 9 \times 4 = \underline{\phantom{0}}\quad 36 \times 1 = \underline{\phantom{0}}\]

   \[48 \div 1 = \underline{\phantom{0}}\quad 48 \div 6 = \underline{\phantom{0}}\quad 48 \div 2 = \underline{\phantom{0}}\]

   \[48 \div 8 = \underline{\phantom{0}}\quad 48 \div 4 = \underline{\phantom{0}}\]

2. Answer these Questions:
   1. Why are the answers to the multiplication problems the same whole number?

2. Why can the same whole number, 48, be divided by different numbers?
Answers

\[
\begin{align*}
6 \times 6 &= \_36\_ & 18 \times 2 &= \_36\_ & 12 \times 3 &= \_36\_ & 9 \times 4 &= \_36\_ & 36 \times 1 &= \_36\_ \\
48 \div 1 &= \_48\_ & 48 \div 6 &= \_8\_ & 48 \div 2 &= \_24\_ & 48 \div 8 &= \_6\_ & 48 \div 4 &= \_12\_ 
\end{align*}
\]

Possible Answers to Class Discussion Questions:

1. **Students may have answers like:** The answers to all of the multiplication problems are the same because different numbers can go into 36. Or students may say that 36 has many different factors.

2. **Students may have answers like:** Different numbers can divide 48 evenly because there are many numbers that go into 48. The number 48 also has many factors.
Name: ________________________________

Clock Activity

A normal clock has 12 dots around the face. Each dot corresponds with a number, which represents the hour in time. Today you will be constructing your own clocks with different numbers of dots. You are also going to be exploring the different ways you can end up back at the start of your clocks. You will be put into groups of 4 or 5. Each of you will complete your own clocks and tables, but HELP each other complete the activity. Remember to HAVE FUN! 😊

Directions:
1. Each of you will be given 4 clock faces with 12, 13, 14, and 15 dots.
2. Designate a dot to be the start dot and label it “start”.
3. In the tables the “jump sizes” on the left column represents the number of dots you jump to get the next one. For example if the jump size is 1 and there are 12 dots, it would take you 12 jumps to reach “start” again.
4. Fill out the right column with the number of jumps it took you to reach start again.
5. You may use different colors to show the jumps on the clocks.
6. If there is not an even number of jumps that allows you to end up at “start”, put NO in the box.
7. After you have finish answer the questions below.

Based on your observations list all the factors of ➔

12:

13:

14:

15:

Challenge: List all the factors of 30.

30:

What is the Greatest Common Factor between 12 & 13? _______

What is the Greatest Common Factor of 12, 15, & 30? _______

What is the Greatest Common Factor of 12, 13, 15, & 30? _______

What is the Greatest Common Factor of 12, 13, 14, 15, & 30? _______
Name: ________________________________

Directions: Fill out the table below with your observations.

Clock #1 – 12 Dots

<table>
<thead>
<tr>
<th>Jump Size</th>
<th>Number of Jumps</th>
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</thead>
<tbody>
<tr>
<td>1</td>
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<td>14</td>
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<td>15</td>
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</tbody>
</table>

Clock #2 – 13 Dots

<table>
<thead>
<tr>
<th>Jump Size</th>
<th>Number of Jumps</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
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<tr>
<td>3</td>
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<td>14</td>
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<td>15</td>
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</tr>
</tbody>
</table>
#1 - 12 Dots

#2 - 13 Dots
#3 – 14 Dots

#4 – 15 Dots
Name: ______________________________________

Closing Activity/Problem

Complete these problems (Don’t forget to show your work!):

1. George wishes to create a rectangular garden in his backyard. He knows that his garden needs to be a total of 36 square feet. List all possible dimensions that George’s garden can have. Or in other words, make a list of factor pairs that fit the description of the garden.

2. George’s next-door neighbor, Sally, wishes to also make a garden. Sally’s garden must be 96 square feet. Make a list of all possible dimensions Sally’s garden can have.

3. The two gardens are going to have to share a common side. The side that is shared must be as long as possible. What is the length of the side that is shared between the two gardens?

4. Explain, in the correct terms, how you solved this problem using the information you learned in the lesson. Show all work leading up to your answer.
Answers

1. 36: 1, 2, 3, 4, 6, 9, 12, 18, 36
   The garden can be:
   36 ft. by 1 ft.
   18 ft. by 2 ft.
   12 ft. by 3 ft.
   9 ft. by 4 ft.
   6 ft. by 6 ft.

2. 96: 1, 2, 3, 4, 6, 8, 12, 16, 24, 32, 48, 96
   The garden can be:
   96 ft. by 1 ft.
   48 ft. by 2 ft.
   32 ft. by 3 ft.
   24 ft. by 4 ft.
   16 ft. by 6 ft.
   12 ft. by 8 ft.

3. 12 feet

4. Students may have answers like:

   I found this problem by listing all the factors of 36 and 96. Since the
   side of the garden had to be as long as possible you are finding the greatest
   common factor of 36 and 96. I looked at each factor list and found out that
   12 is the GCF. So the side that is shared by the two gardens must be 12 feet
   long.
Exit Slip
Please answer the following questions:

4. Explain in your own words the terms factor and greatest common factor.

5. Explain or show how you would find the greatest common factor between 50 and 45.

6. List at least one concept or topic from today’s lesson that you are confused about or do not understand completely.

Students may have answers like:

1. A factor is a whole number that can be multiplied by another whole number to equal a whole number product. A Greatest Common Factor is the greatest factor between two or more whole numbers have in common.

2. To find the GCF of 50 and 45 I would make a factor list for each.
   45: 1, 3, 5, 9, 15, 45
   50: 1, 2, 5, 10, 25, 50
   The GCF is 5 because it is the greatest factor 50 and 45 have in common.

3. One concept I am confused and don’t understand is how to find the GCF of more than two numbers. I am also confused of how to find the factors of larger numbers.

   As a teacher this would tell me that maybe the next day when going over homework, practicing finding GCF’s of more than two numbers and factors of larger numbers would be necessary to solidify the concepts learned in the lesson.
Factors and GCF Homework

Complete the following problems. Make sure to show your work! 😊

Find all the factors of each number.
1. 12:
2. 27:
3. 45:
4. 53:
5. 64:
6. 71:
7. 100:
8. 144:

Find the GCF of each group of numbers.
1. 9, 12 ____
2. 14, 21 ____
3. 64, 48 ____
4. 60, 75, 90 ____
5. 48, 120 ____

Answer the following question.
1. Can two EVEN numbers have a greatest common factor of 1? Why or why not?
Homework Answers

Find all the factors of each number.
1. 1, 2, 3, 4, 6, 12
2. 1, 3, 9, 27
3. 1, 3, 5, 9, 15, 45
4. 1, 53
5. 1, 2, 4, 8, 16, 32, 64
6. 1, 71
7. 1, 2, 4, 5, 10, 20, 25, 50, 100
8. 1, 2, 3, 4, 6, 8, 9, 12, 16, 18, 24, 36, 38, 72, 144

Find the GCF of each group of numbers.
1. 3
2. 7
3. 16
4. 15
5. 24

Answer the following question.

1. Can two EVEN numbers have a greatest common factor of 1? Why or why not?

No. It is impossible for two even numbers to have a GCF of one because even numbers always have a factor of 2. The GCF between 2 even numbers must be at least 2.